According to this model, the nuclei of all elements behave like a *liquid drop* of electrically charged, incompressible *liquid* of constant density but of varying mass. The 'liquid' in reality consists of neutrons and protons.

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbours, like the molecules of a liquid which ideally are free to move about but always maintain a fixed intermolecular distance from their nearest molecules.

Assumptions. The main assumptins of liquid drop model are:

(*i*) The nucleus is supposed to be spherical in shape in its stable state just as a liquid drop is spherical due to the symmetrical forces of surface tension.

(*ii*) The volume as well as the mass of the nucleus being proportional to the atomic mass number *A*, the density of a nucleus is constant, independent of volume. Similarly, the density of a liquid drop is constant, independent of its volume. [One point of difference should, however, be noted. The density of a nucleus is the *same* for nuclei of *different* elements but the density of a liquid is different for different liquids.]

(*iii*) The nucleons are supposed to move about within a spherical enclosure repersented by the nuclear potential barrier just as the molecules move about within a spherical drop of the liquid, the shape of which is determined by properties of surface tension.

(*iv*) The two main properties of nuclear forces are their short range and tendency to saturate. These can be deduced from the fact that (*a*) The binding energy per nucleon is almost a constant quantity (nearly 8 MeV) for all but the lighter nuclei and, therefore, the total binding energy of the nucleus is proportional to the total number of nucleons *i.e.*, its mass number *A* and (*b*) the volume of the nucleus is also proportional to the atomic mass.

(*v*) Just as the latent heat of vaporisation is a constant for a liquid, the binding energy per nucleon is also a constant.

(*vi*) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is fomed which emits nuclear radiations almost immediately.

(*vii*) When a small drop of a liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.

Justification. From the above assumptions we find that the properties of a nucleus are very similar to the properties of a liquid drop. Hence it is justified to call this model of the nucleus as *liquid drop model.*

(*b*) Expression for binding energy. The *mass M* of a neutral atom whose nucleus contains *Z* protons and $N = (A - Z)$ neutrons is given by $M(Z, A) = Zm_p + (A - Z)\, m_n - E_b$

or $$M = Zm_p + Nm_n - E_b$$

where E_b is the binding energy. The total binding energy of a nucleus is contributed by the following factors.

1. Volume energy. Suppose the energy associated with each nucleon-nucleon bond has a value *U*. The energy is really negative because attractive nuclear forces are involved but is usually taken as positive for convenience. Because each bond energy *U* is shared by two nucleons, each has a binding energy $\frac{1}{2}U$. When an assembly of spheres of the same size is packed together into the smallest volume each interior sphere has 12 other spheres in contact with it. This is the situation within the nucleus where each interior nucleon has 12 other nucleons in contact with it and hence has a binding energy $12 \times \frac{1}{2}U = 6U$. If all the *A* nucleons in the nucleus are supposed to be in the interior, the total binding energy of the nucleus would be

$$E_v = 6UA = a_v A$$

where a_v is a constant having a value 14 MeV. This energy is known as *volume energy.*

2. Surface energy. The nucleons on the surface of the drop interact only with nucleons on one side. The above expression which has been derived on the assumption that all nucleons are being equally attracted on all sides, has to be reduced by a factor proportional to the number of nucleons on the surface. The radius of the nucleus is given by

$$R = r_0 A^{1/3}$$

$\therefore$ Surface area $= 4\pi R^2 = 4\pi r_0^2 A^{2/3}$

$\therefore$ Surface energy $E_s = -k\,4\pi r_0^2 A^{2/3} = -a_s A^{2/3}$

where a_s is a constant = 13 MeV

The negative sign indicates that this energy contribution is in a direction opposite to the volume energy, becasue the nucleons on the surface are not as strongly bound as the nucleons inside. Since the relative number of surface nucleons is pretty high for light nuclei, the surface energy factor is quite large in their case.

3. Coulomb energy. A nucleus contains Z protons which can form $\frac{Z(Z-1)}{2}$ pairs. The electronic repulsion between each pair of protons in the nucleus contributes towards decreasing of its binding energy. The energy due to repulsion in protons is known as Coulomb energy. Thus, the Coulomb energy E_C of a nucleus is the work that must be done to bring together Z protons from infinity into a spherical aggregate the size of the nucelus. The potential energy for a pair of protons a distance r apart.

$$U = \frac{e^2}{4\pi\varepsilon_0 r}$$

As there are $\frac{Z(Z-1)}{2}$ proton pairs,

$$E_C = \frac{Z(Z-1)}{2}\frac{e^2}{4\pi\varepsilon_0 r} = \frac{Z(Z-1)e^2}{8\pi\varepsilon_0}\left(\frac{1}{r}\right)_{av}$$

where $\left(\frac{1}{r}\right)_{av}$ is the value of $\frac{1}{r}$ averaged over all the pairs. If the pairs are uniformly distributed throughout the nucleus of radius R, then $\left(\frac{1}{r}\right)_{av} \propto \frac{1}{R}$ and hence proportional to $\frac{1}{A^{1/3}}$

$\therefore$ Coulomb energy $E_C \propto \frac{Z(Z-1)e^2}{8\pi\varepsilon_0}\frac{1}{A^{1/3}}$

$$= -a_c \frac{Z(Z-1)}{A^{1/3}}$$

[For detailed proof **See Q. 23.6**]

where a_c is an unkown constant = 0.60 MeV. The Coulomb energy is negative because it arises from an effect which opposes nuclear stability.

4. Asymmetry energy. In general, in heavy nuclei, the number of neutrons $N > Z$ the number of protons. In light nuclei, $N = Z$ and these nuclei are highly stable. As the number of neutrons increases, the nucleus acquires an asymmetrical character due to which a force comes into play which reduces the volume energy. This reduction in energy is directly proportional to the square of the excess of neutrons over protons and inversely proportional to the total number of nucleons.

$$\therefore \quad E_a \propto \frac{(N-Z)^2}{A} = -a_a \frac{(N-Z)^2}{A} = -a_a \frac{(A-2Z)^2}{A} \qquad \text{[For proof See Q. 23.5]}$$

where a_a is a constant $= k(a_v - a_s A^{1/3})$ and $k = 1.7826$

The value of a_a can be taken to be 19 MeV for most of the cases.

5. Pairing and shell energy. Nucleons have a tendency to exist in pairs. Hence the nuclei with even number Z of protons and even number N of neutrons are highly stable, even Z-odd N, or odd Z-even N, are less stable. The nucelons in a nucleus exist in the form of shells which may be filled by a certain number given by relevant selection rules. Taking both the effects together the term contributing towards volume energy is given by

$$E_p = (\pm, 0)\, a_p A^{-3/4}$$

where a_p is a constant and has a value – 34 MeV for even Z-even A nuclei, *zero for odd A-odd N* nuclei and + 34 MeV for even A and odd Z nuclei.

Weizacker's semi-empirial formula. Combining all the terms which contribute towards the binding energy of the nucleus, we get Weizacker's semi-empirical binding energy formula which can be written as

Total binding energy $E_b = E_v + E_s + E_c + E_a + E_p$

$$= a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + (\pm,0)\, a_p A^{-3/4}$$

The *binding energy per nucleon* is given by

$$\frac{E_b}{A} = a_v - \frac{a_s}{A^{1/3}} - a_c \frac{Z(Z-1)}{A^{4/3}} - a_a \frac{(A-2Z)^2}{A^2} (\pm,0)\, a_p \frac{A^{-3/4}}{A}$$

Semi-empirical mass formula. The complete semi-empirical mass formula on the liquid drop model is

$$M(Z, A) = Zm_p + (A-Z)\, m_n - E_b$$

$$= Zm_p + (A-Z)\, m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} (\pm, 0)\, a_p A^{-3/4}$$

Graph. The contribution of various energies (except pairing and shell energy E_p) per nucleon towards the binding energy of the nucleus is shown in Fig. 23.1.

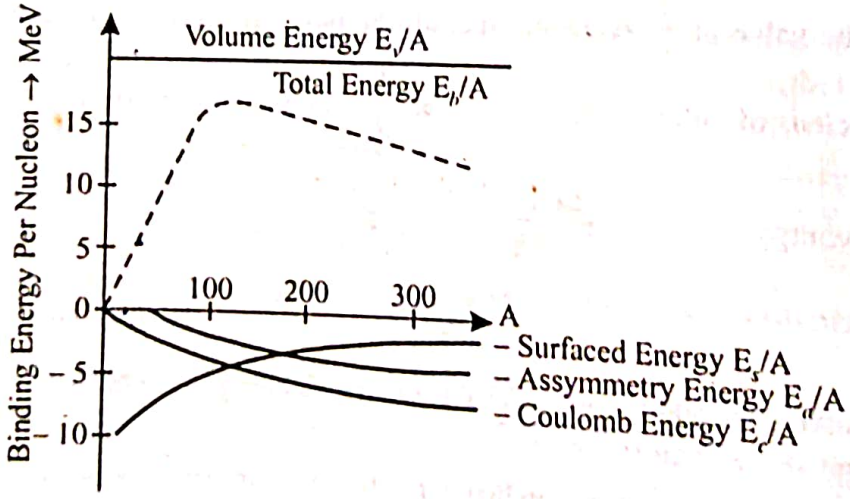

Fig. 23.1.

(c) Achievements (utility). (i) Stable nucleus. We can explain the stability of a nucleus o the liquid drop model. The stability of a liquid drop is due to the forces of cohesion between th molecules. Similarly, the stability of a nucleus is due to the binding energy of each nucleon. Just a

to remove a molecule from a liquid drop energy has to be supplied to it in the form of heat, energy will have to be supplied to the nucleon equal to or greater than its binding energy to remove it. Hence, the stability of the nucleus.

(*ii*) Radioactive nucleus. In a liquid, with the increase of temperture, the thermal agitation of its molecules becomes more and more rapid and at a particular stage evaporation may take place. A molecule, in a liquid drop, evaporates by gaining energy from its neighbouring molecules during the process of collisions. Similarly, a nucleon or a group of nucleons may leave the nucleus by gaining energy from the neighbouring nucleons during the process of collision, thus exhibiting the phenomenon of radioactivity.

(*iii*) Artificial radioactivity. The liquid drop model explains the phenomenon of *artificial radioactivity*. It is supposed that when a nucleus is bombarded by fast moving particles, an incoming particle enters the target nucleus forming a *compound nucleus*. It quickly shares its energy with the nucleons already present so that no single particle has sufficient energy for escape. The decay or disintegration of the compound nucleus occurs when the energy is again accidentally concentrated on some one particle which escapes giving rise to the phenomenon of artificial radioactivity, or the energy may by lost by emission of a γ-ray.

(*iv*) Fission. The liquid drop model also explains the phenomenon of *nuclear fission*. For detailed explanation see **Q. 25.32** (Chapter **25**).

(*v*) The liquid drop model helps us to calculate the atomic masses and binding energies of the nuclei quite accurately.

Failures (limitations). (*i*) The liquid drop model fails to explain the high stability of nuclei with magic numbers.

(*ii*) This model does not explain the measured spins and magnetic moments of the nuclei.

Q. 23.3. Calculate the atomic number of the most stable nucleus for a given mass number A on the liquid drop model. Hence explain why out of $_2He^6$, $_4Be^6$ and $_3Li^6$ only the last one is stable.

Ans. Stable nucleus. According to the liquid drop model the binding energy E_b is given by

$$E_b = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + a_p A^{-3/4}$$

Putting $Z(Z-1) = Z^2$ as 1 can be neglected as compared to Z, we get,

$$E_b = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + a_p A^{-3/4} \quad \ldots (i)$$

The most stable nucleus for a given mass number A is that which has the maximum value of binding energy *i.e.*, for which

$$\left(\frac{\partial E_b}{\partial z}\right)_{A \text{ constant}} = 0$$

From (*i*), we have

$$\frac{\partial E_b}{\partial z} = -2a_c A^{-1/3} Z + 4a_a (A-2Z) A^{-1} = 0 \quad \ldots (i)$$

or $$4a_a - 8a_a A^{-1} Z = 2a_c A^{-1/3} Z$$

or $$Z(4a_a + a_c A^{2/3}) = 2a_a A$$

$$Z = \frac{A}{2 + \frac{a_c}{2a_a} A^{2/3}}$$